

# A Novel Study on Implementation of Adaptive Filtering using Normalized LMS with Different Targets

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**Abstract** – This paper proposes a study on adaptive filtering response using normalized LMS (Least mean square) algorithm. NLMS algorithm has low computational complexity and good convergence speed. It has minimum steady state error. Here it uses three different target filters FIR, IIR and multiband Equiripple filters. Also covers the effects of stationary signals on the performance of adaptive filters. A detail study of this filter is done by taking into account different cases. The effects of changes in parameters were noted within a specific filter and later a comparison between the filters was done. Noise variance was another factor that was considered to learn its effect. Also parameters of adaptive filter, such as step size and filter order, were varied to study their effect on performance of adaptive filters. The results achieved through these test cases are discussed in detail and will help in better understanding of adaptive filters with respect to signal type, noise variance and filter parameters. The first test led us to conclude the step size increases the convergence speed of a filter from transient to steady state but at the same time increase the error variation in the steady state.

**Index Terms** – Adaptive Filters, NLMS filters, LMS filters, Different targets in adaptive filters.

## 1. INTRODUCTION

Real world signals are analogy and continuous, eg: an audio signal, as heard by our ears is a continuous waveform which derives from air pressure variations fluctuating at frequencies which we interpret as sound. However, in modern day communication systems these signals are represented electronically by discrete numeric sequences. In these sequences, each value represents an instantaneous value of the continuous signal.

DSP (Digital signal processing) is one of the technical fields that demands high speed and low power digital filters. Digital filter is very important class of linear time invariant system that is used to remove unwanted signal such as noise or echo signal. Digital filter is used because it has advantages over analog filter such as easier storage and

maintenance, higher flexibility and minimum effect of interference noise [1].

The designing of digital filter requires the approved specification with fixed coefficients. If the specification is time varying or not accessible then this problem can be manipulated by digital filter with adaptive coefficients, which is known as adaptive filter. To design Adaptive filters, LMS, NLMS and RLS algorithm is used.

Linear filtering is required in a variety of applications. A filter can be considered as optimal only if it is designed with some knowledge about the input data. Adaptive filters are used when this information is not known. The adjustable parameters in the filter are assigned with values based on estimated statistical nature of the signals. So, these filters are adaptable to the changing environment [2].

Many other algorithms have been developed based on Linear Programming (LP), Quadratic Programming and Heuristic methods in Artificial Intelligence (AI). Remez Exchange Algorithm (to design equiripple filter) and linear Programming (to design adaptive filter) are optimum in the sense that these methods achieve both a given discrimination and a specified selectivity with a minimum length of the filter impulse response. In contrast to the FIR filter, the infinite impulse response (IIR) filter, as its own name suggests has infinite impulse response. the output sample,  $y(n)$ , depends on past outputs samples,  $y(n-k)$ , also on present and past inputs samples,  $x(n-k)$ , that is known as the IIR filters feedback. The strength of the IIR filters comes from that feedback procedure, but the disadvantage of it is that the IIR filter becomes unstable or poor in performance if it is not well designed [3].

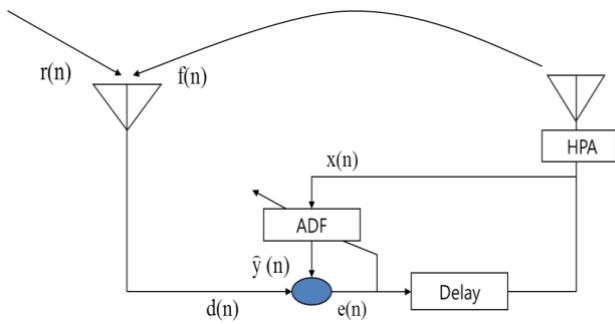


Figure 1: Interference Cancellation System using Adaptive Filters

Some had discussed the comparison between adaptive filtering algorithms that is least mean square (LMS), Normalized least mean square (NLMS), Recursive least square (RLS). Execution aspects of these algorithms, their computational complexity and Signal to Noise ratio are examined. Here, the adaptive behavior of these algorithms is analyzed. Recently, adaptive filtering algorithms have a nice trade-off between the complexity and the convergence speed. Efficient utilization of limited radio frequency spectrum is only possible to use smart antenna system. LMS and NLMS algorithms are beam forming algorithms. Smart antenna uses these algorithms in coded form which calculates complex weights according to the signal environment [4].

This paper is organized as follows. In section 2<sup>nd</sup>, we describe the adaptive filter. In section 3<sup>rd</sup>, it describes the proposed work. In section 4<sup>th</sup>, it describes the adaptive filter algorithm for analysis the Digital filter and measure performance in terms of parameters. Finally conclusion is given in section 5<sup>th</sup>.

## 2. DESCRIPTION OF ADAPTIVE FILTERS

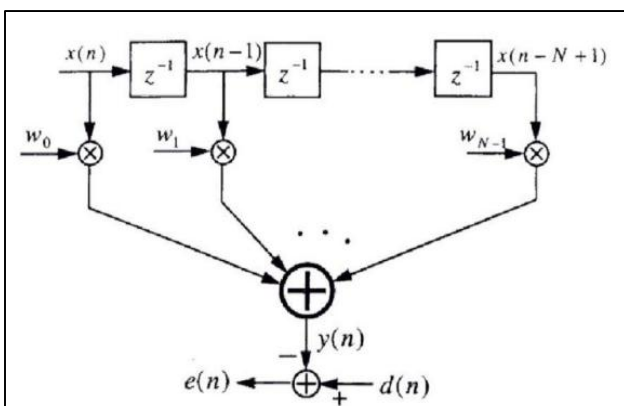


Figure 2: General Block Diagram of Adaptive Filter [11]

Adaptive filtering is properly used due to its esteemed knowledge of signal makeup, so signal analysis is related to the adaptive processing. Literally, the word ‘adaptive’ means to adjust with other environment (system) by having the same

response as the system itself to some phenomenon which is taking place in its surroundings. The systems which carries out its functionality after undergoes the process of adaptation is called filter. The term ‘filter’ means to take the unnecessary particles (frequency component) from its input signal and process them to generate required output under certain specific rules. Adaptive filters are dynamic filters which iteratively alter their characteristics in order to achieve an optimal desired output [5].

Adaptive filter use the algorithm by which itself adjust the transfer function. It enables the filter to produce an output which is same as the output of an unknown system. It removes the problem of Weiner filter. It is totally based on stochastic approach. Adaptive filters works on the principle of minimizing the mean square difference that is, error between the filter output and designed signal. The error signal can be generated by the output of the programmable variable coefficient digital filter subtracted from a reference signal [10].

Adaptive filters are made up of FIR and IIR filters. FIR adaptive filters are mostly used due to the stability for any set of fixed coefficient. The algorithms for adjusting the coefficient of FIR filter are simpler in general than those for adjusting the coefficients of IIR filter. Adaptive filtering can be classified into three categories: adaptive filter structures, adaptive algorithms, and applications. The performance of the adaptive algorithm is important for all systems; it is also essential how adaptive system is functioning. The choice of algorithm is highly dependent on the signals of interest, the operating environment, as well as the convergence time required and computation power available. For any application the adaptive algorithm provide competent performance evaluations for the structures of various filter and adaptive algorithm. An adaptive digital filter can be built up using an IIR (Infinite impulse response) or FIR (Finite impulse response) filter. Adaptive FIR filter structure is the transversal filter which implements an all-zero filter with a canonic direct form (without any feedback). FIR is naturally stable because its structure involves only forward paths and no feedback exists. The presence of feedback to the input may lead the filter to be unstable and can occur. The output can be represented by linear combination of the adaptive filter coefficients for this adaptive FIR filter. Alternative adaptive FIR filter improves performance in terms of convergence speed [6].

An adaptive filter is a computational device that attempts to model the relationship between two signals in real time in an iterative manner. In an iterative manner means that the parameters need to adjust continuously. For example, adaptive filter needs the output signal to be alike the input signal in a manner that its least mean square error is minimized. In this case, the adaptive algorithm needs to be in an iterative manner [7].

An adaptive filter can be defined by four aspects:

1. The signals which are processed by the filter
2. The structure to characterize how the output signal of the filter is computed from its input signal
3. The parameters within the structure that can be iteratively changed to change the filter's input-output relationship.
4. The adaptive algorithm to identify how the parameters are adjusted from one time instant to the next.

As the NLMS is an extension of the standard LMS algorithm, its practical implementation is very similar to that of the LMS algorithm except that the NLMS algorithm has a time varying step size  $\mu(n)$ . This step size can improve the convergence speed of adaptive filter. Each iteration of the NLMS algorithm requires these steps in the following order [11]. The only difference with respect to LMS is the coefficient updating step (4).

### 2.1. LMS Algorithm

Three steps are involved in every iteration of LMS algorithm [8] as:-

1. The output of the FIR filter,  $y(n)$  is calculated using equation

$$y(n) = w^T(n)x(n) \quad (1)$$

2. The value of the error estimation is calculated using equation

$$e(n) = d(n) - y(n) \quad (2)$$

3. The tap weights of the FIR vector are updated for next iteration by equation

$$W(n+1) = w(n) + 2\mu e(n) x(n) \quad (3)$$

LMS algorithm is most widely used due to its computational simplicity. It has the fixed step size with upper bound and lower bound as

$$0 < \mu < 2/\lambda_{\max}$$

### 2.2. NLMS Algorithm

The main limitation of LMS algorithm is that it is sensitive to scaling of its input  $x(n)$  which makes it hard to select a step size  $\mu$  that makes stability of the algorithm. NLMS (normalized least mean square) algorithm also is modified form of LMS algorithm by normalizing with power of input with time varying step size. In each iteration of the NLMS [9] algorithm requires three steps in the following order:-

The output of the adaptive filter is calculated.

$$y(n) = w^T(n) x(n) \quad (4)$$

1. An error signal is calculated as the difference between the desired signal and filter output

$$e(n) = d(n) - y(n) \quad (5)$$

2. The step size value is calculated from the input vector
3. The filter tap weights are updated in preparation for the next iteration

$$w(n+1) = w(n) + 2\mu e(n) x(n) \quad (6)$$

NLMS algorithm has greater stability with unknown signals. It has also good convergence speed and relative computational simplicity.

### 3. DESCRIPTION OF PROPOSED SYSTEM

The main problem in this work is convergence of filters at higher orders and step size. The presented work proposes an adaptive FIR filter design using normalized least mean square algorithm (NLMS) and also compares its results for various filters like FIR, IIR and multiband FIR at different orders. It has been observed from the literature study that the implementation of FIR filter using the filter coefficients is computationally very expensive on account of floating point arithmetic's i.e. no. of adders, multipliers and shifting operations. However, if the filter coefficients are evaluated using the normalized least mean square algorithm towards the maximum round off and convolved with the input signal for speedy operation of the same.

The main reason for the LMS algorithm's popularity in adaptive filtering is its computational simplicity, making it easier to implement than all other commonly used adaptive algorithms. For each iteration, the LMS algorithm requires  $2N$  additions and  $2N+1$  multiplication.

The main drawback of pure LMS algorithm is that it is sensitive to the scaling of its input  $x(n)$  which makes it hard to choose a step sizes  $\mu$  that guarantees stability of the algorithm. The Normalized least mean square filter (NLMS) is a variant of LMS algorithm that solves the above problem by normalizing with the power of input. In other sense, we can say that normalized LMS (NLMS) algorithm is a modified form of the standard LMS algorithm [15]. To reduce the computational complexity and convergence time or increase convergence speed, some alternative LMS based algorithms are used. These are quantized-error algorithm, LMS-Newton algorithm, Normalized LMS algorithm, frequency-domain algorithm; affine projection algorithm. But in our present thesis work, main focus is on NLMS algorithm. The Normalized LMS algorithm utilizes a variable convergence factor that minimizes the instantaneous error. Such a convergence factor usually reduces the convergence time but increases the miss-adjustment.

Block diagram of proposed system is given in fig. 3. Here we first take a sinusoidal input signal  $x(n)$  that is combined with desired response to give output  $d(n)$ . This desired response is the output of different target filters used in our designing process. These target filters are equiripple filter, FIR filter, Butterworth IIR filter. The desired response  $d(n)$  will combine with the adaptive filter response  $h(n)$  to give final output. Then we will find the error in the output signal.

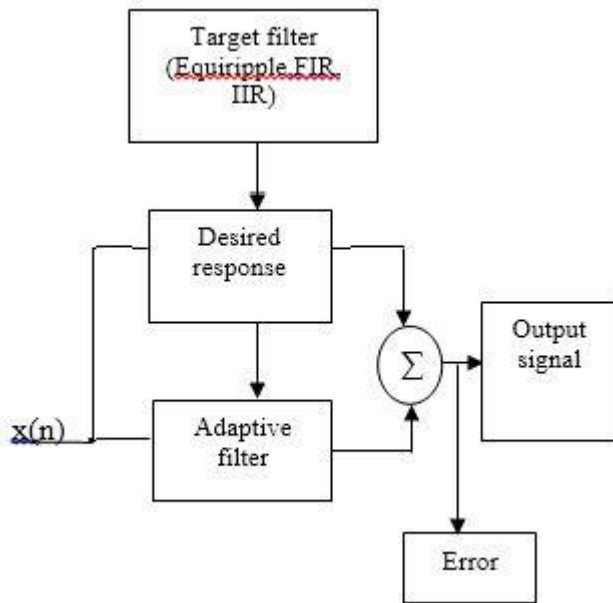


Figure 3: Block Diagram of Proposed System

In our present thesis work we design our filter by using only hanning window. In Frequency Sampling Method, we specify the desired frequency response  $H_d(w)$  at a given set of equally spaced frequencies at  $N$  samples. This method used to design non prototype filters where desired magnitude can be in any irregular shape. As the above methods can not accurately control border frequencies of pass band and stop band in the practical application and are based on fixed formulation and not iterative as a result, many researchers have presented some optimal design approaches. NLMS algorithm has many advantages over LMS like low computational complexity, good convergence speed and minimum steady state error.

For our present research work, we follow the following steps:-

- First of all, different type of filter designing techniques and their related algorithms mainly LMS and NLMS are studied.
- Then we design a standard NLMS adaptive filter with different step size and different frequencies in MATLAB.
- We design target multiband equiripple filter with desired design parameters in MATLAB and compare its performance with NLMS adaptive filter designed in step2. To set these design parameters of given filter, we design a code in MATLAB.
- Then we design Butterworth IIR band pass filter with different design parameters in MATLAB and compare its performance with NLMS adaptive filter as above.
- In the similar way, we design Low pass FIR filter using Hanning window with different design parameters in MATLAB and compare its performance with NLMS adaptive filter.

- For all the above designed filters i.e. equiripple filter, IIR and FIR filter, we analyze its magnitude response and phase response and compare their magnitude response and phase response with NLMS adaptive filter.
- Then we do the error analysis of all the target filters individually and finding which filter is better for designing purpose.
- At last comparison is done between LMS and NLMS on the basis of error plot.
- Basically, we have done MATLAB simulations for the entire dissertation work, right from designing of Equiripple FIR filter, Butterworth IIR filter and FIR filter using hanning window as target filters and comparing their performance with designed NLMS Adaptive filter. Then we design code using MATLAB for finding the error in all the filters. Change in several parameters like step size, filter order, sampling frequency is done so as to minimize the error and convergence time.

### 3.1. Performance Parameters

#### RMSE:

It is a measure of the difference between value predicated by estimation and value actually observed from the thing being estimated.

#### Convergence Rate:

The convergence rate is defined as the number of iterations required for the algorithm to converge to its steady state mean square difference that is error.

#### Complexity:

Computational complexity is the measure of the number of arithmetic calculation like Multiplications, addition and subtraction for different adaptive algorithm.

## 4. SIMULATION RESULTS

### 4.1. NLMS with Equiripple Target

| Design Parameters         | Value                                     |
|---------------------------|---|
| Response type             | Multiband                                 |
| Design method             | Equiripple FIR                            |
| Filter order              | 98 (Length=99)                            |
| Freq. vector (Normalized) | [0,0.28,0.3,0.48,0.5,0.69,0.7,0.8,0.81,1] |
| Magnitude vector          | [0,0,1,1,0,0,1,1,0,0]                     |
| Weight vector             | [1,1,1,1,1]                               |
| Sampling frequency        | 8000                                      |
| Modulating frequency      | 8000                                      |
| Step size                 | 1.4                                       |

Table 1: Design Parameters of Multiband Equiripple Filter

In this, it uses a sinusoidal input to generate a desired signal using Multiband Equiripple as a target filter. After selecting the

filter type then it has to give value of the step size ( $\mu$ ) and filter order ( $N$ ) which are required for the algorithm simulation. The filter length used is 99. The step size determines the updating speed of filter coefficients. Here, step size is 1.4. For designing of equiripple filter as the target filter, fix values of different parameters. Also, Parameters having same values are used for designing of adaptive NLMS filter.

Comparing the responses between the converging result and the target, the amplitude and phase responses of both filters are drawn in the same figure of Figure. In this fig, magnitude response of target filter & adaptive NLMS almost overlap with each other, while phase response is completely overlapping each other at different normalized frequencies. Their responses are approximated to ideal filter response.

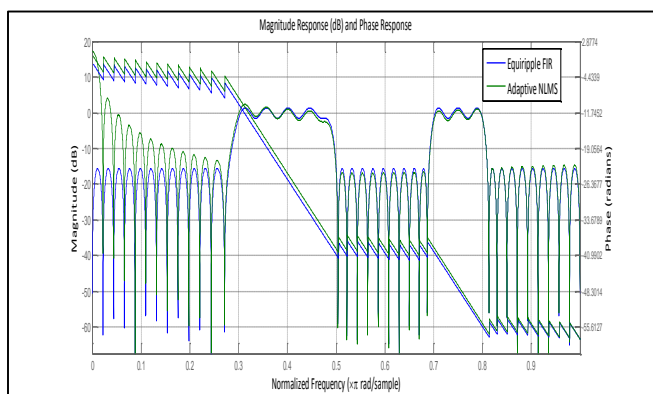


Figure 4: Response of NLMS with Equiripple Target

#### 4.2. NLMS with FIR Target

In this, sinusoidal input is used to generate a desired signal using FIR as a target filter. To get desired response, it uses a NLMS adaptive filter with a length of 19, and the target FIR filter is with a length of 21. Windowing functions are most easily understood in the time domain; however, they are often implemented in the frequency domain instead. FFT windows reduce the effects of leakage but cannot eliminate leakage entirely.

| Parameters           | Value for Target FIR filter | Value for Comparing FIR filter | Value for NLMS Adaptive filter |
|----------------------|-----------------------------|--------------------------------|--------------------------------|
| Response type        | Low pass                    | Low pass                       | Low pass                       |
| Window Used          | Hanning                     | Hanning                        | Hanning                        |
| Filter length        | 21                          | 19                             | 19                             |
| Sampling frequency   | 30                          | 30                             | 30                             |
| Modulating frequency | 15                          | 15                             | 15                             |
| Step size            | 1.4                         | 1.4                            | 1.4                            |

Table 2: Design Parameters of NLMS with FIR Filter

In effect, they only change the shape of the leakage. In addition, each type of window affects the spectrum in a slightly different way. Hanning window has good frequency resolution, less spectral leakage and good amplitude accuracy. That's why it is preferred over other windows.

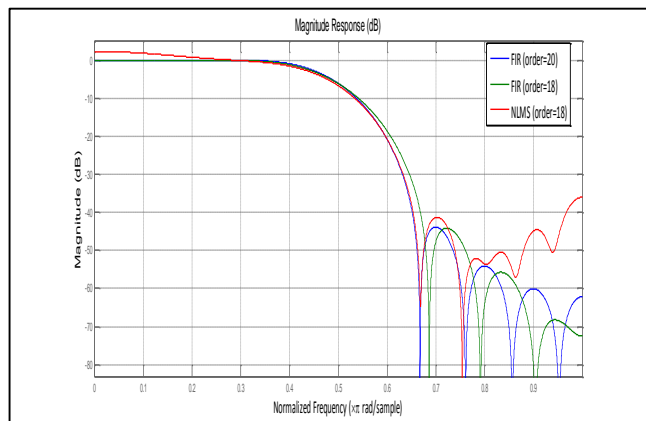


Figure 5: Response of NLMS with FIR Target

The fig 5 shows the response of proposed NLMS filter of order 18. It has low stop-band attenuation and does not have a flat response. It has response of sine wave. As shown in Fig, the proposed converging result has comparable roll-off sharpness and with the larger order FIR filters which are better than the comparing same order FIR filter. The filter response shows that it is trying to extract a single frequency component as it was required and expected.

#### 4.3. NLMS with IIR Target

The primary advantage of IIR filters over FIR filters is that they typically meet a given set of specifications with a much lower filter order than a corresponding FIR filter. In this, it designs Butterworth Band pass IIR filter as the target filter and also design NLMS adaptive filter in MATLAB. In this, Butterworth IIR filter is chosen as a target filter. Because an IIR filter can achieve a similar amplitude response with a much lower order than its FIR counterpart, and when the NLMS adaptive filter achieves something beyond the edge its structure or length can provide, some instability shall occur, a proper (small) length IIR filter should be chosen to test.

| Parameters                | Value for Target IIR Filter | Value for NLMS Adaptive filter |
|---------------------------|-----------------------------|--------------------------------|
| Response type             | Bandpass                    | Band pass                      |
| Design method             | Butterworth IIR             | Butterworth IIR                |
| Filter order              | 10                          | 98                             |
| Normalized frequency(Fc1) | 0.3                         | 0.3                            |
| Normalized frequency(Fc2) | 0.7                         | 0.7                            |
| Sampling frequency        | 8000                        | 8000                           |
| Modulating frequency      | 4000                        | 4000                           |

|           |      |      |
|-----------|------|------|
| Step size | 1.45 | 1.45 |
|-----------|------|------|

Table 3: Design Parameters of NLMS with IIR Filter

To test the efficiencies of the NLMS adaptive filter modelling a IIR filter with a smaller length, a 10th order Butterworth band pass IIR filter is chosen as the target filter, the length of the NLMS adaptive filter is 99. For a band pass filter, specified Weight as a two-element vector containing the pass band edge frequencies. This allows for a non-causal, zero-phase filtering approach, which eliminates the nonlinear phase distortion of an IIR filter. The Butterworth filter provides the best Taylor Series approximation to the ideal filter. That's why it is preferred over others.

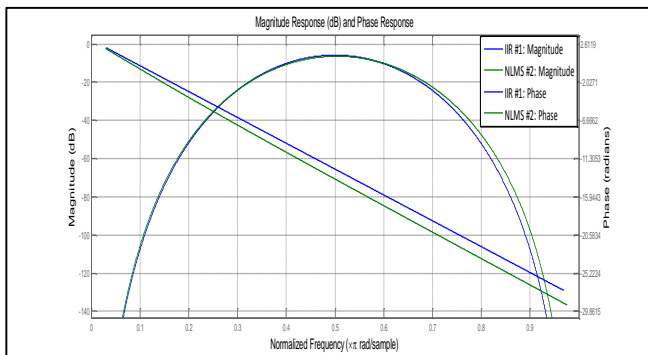


Figure 6: Response of NLMS with IIR Target

The poles of the converging results are always centered at the Centre of the circle. IIR filters can be implemented this way, but only if a floating point processor is available. If using a fixed point processor, an IIR filter must be implemented as a series of second order sections. Pole-Zero plots is an important tool, which helps us to relate the Frequency domain and Z-domain representation of a system. Understanding this relation will help in interpreting results in either domain. It also helps in determining stability of a system. The pole-zero plots gives us a convenient way of visualizing the relationship between the Frequency domain and Z-domain. For a system to be stable, its impulse response must be absolutely summable i.e. lies within range less than 1.

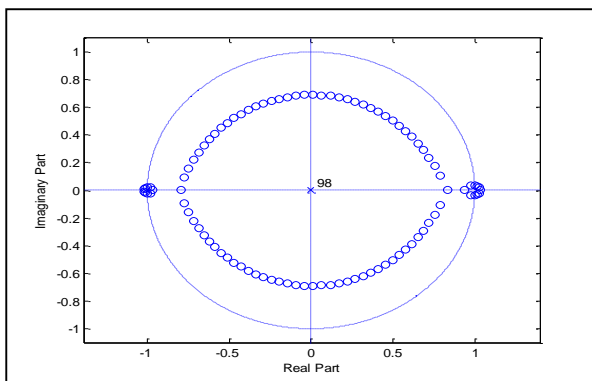


Figure 7: Pole-Zero Plots of Adaptive NLMS

#### 4.4. Effect of Change in Parameters of NLMS Filter

In this case the sampling frequency is 8000 Hz, the filter order at 98, filter type is NLMS and three different step sizes of 0.01, 0.1 and 1.4 are used. The results are shown below. At step size = 0.01, there is large difference in magnitude between target & adaptive filter. Phase difference between them is so large that cannot be seen.

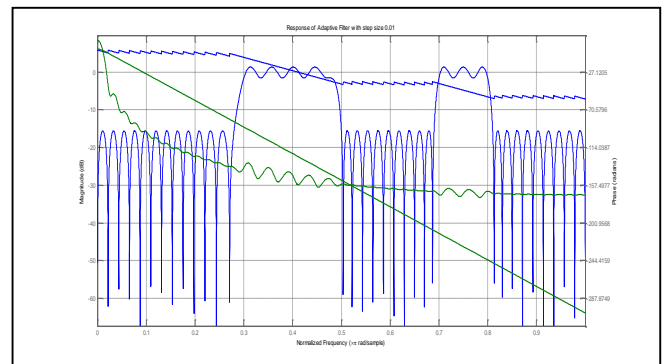


Figure 8: NLMS with Equiripple Target at Step Size=0.01

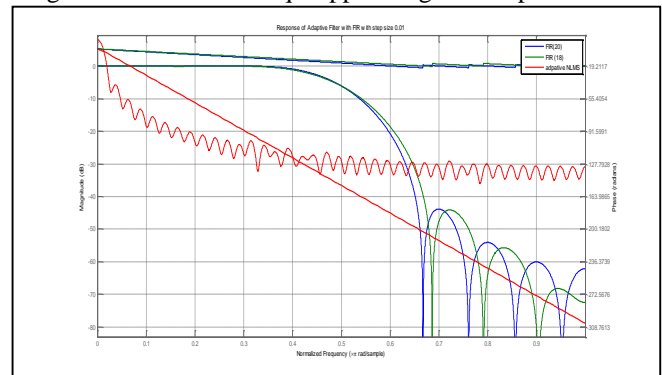


Figure 9: NLMS with FIR Target at Step Size=0.01

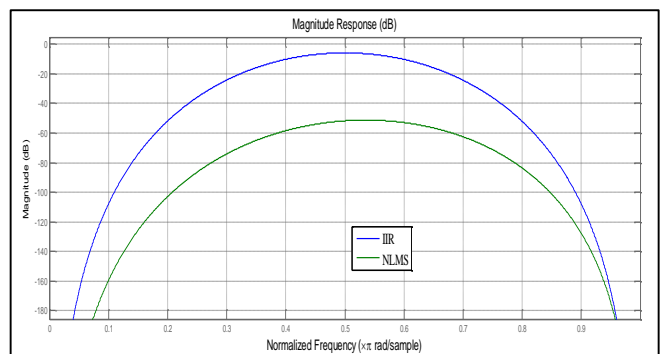


Figure 10: NLMS with IIR Target at Step Size=0.01

#### 4.5. Comparison of NLMS & LMS Filter

In this case the signal length is taken as 19, the step size used is 0.01 and the 18th filter order is used. The results are shown by error plot of LMS & NLMS Filter.



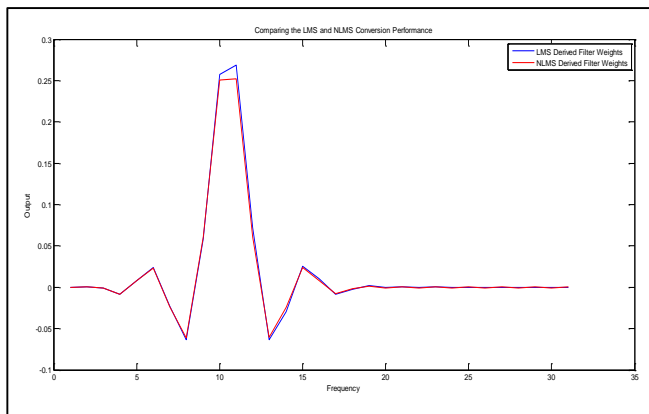


Figure 11: Error Plot of NLMS & LMS Filter

Figure shows the error plot, the smaller step size results in the slow convergence rate at the start but as soon as it enters the steady state we found that smaller step size gives good result by giving less variation. This shows that the smaller step size approaches steady state late but has a good response in steady state. The error plot is the difference of desired signal  $d(n)$  and filter output  $y(n)$ . This difference tells us how close is the filter in producing the desired signal, lower the absolute value of error closer the output of the filter gets to the desired signal. The algorithm LMS and NLMS are also designed and updated according to this error value. The error plot gives us an idea how well the filter is performing.

## 5. CONCLUSION

In this thesis, it proposes the designing and implementation of adaptive filter. The adaptive filter used is NLMS. Here it uses three different target filters FIR, IIR and multiband Equiripple filter. Also covers the effects of stationary signals on the performance of adaptive filters. We tested the signal with variation in step size, filter order and sample frequency. The effects of changes in parameters were noted within a specific filter and later a comparison between the filters was done. The first test led us to conclude the step size increases the convergence speed of a filter from transient to steady state but at the same time increase the error variation in the steady state. The second test on the filter order helped us to see how the filter frequency response varies with variation in filter order. The filter order can be set according to the expectation of the final filter. If the error minimization requirement is less strict we are allowed to keep the filter order low. NLMS performs much better than the LMS for the non-stationary signal that is much more difficult to handle.

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